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A New Illusion of Projected Three-Dimensional Space

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SUMMARY

When perspective projections of orbital trajectories plotted in local-vertical local-horizontal coordinates are viewed with certain viewing angles, their appearance becomes perceptually unstable. They often lose their trochoidal appearance and reorganize as helices. This reorganization may be due to the viewer's familiarity with coiled springs.

INTRODUCTION

Planar projections of three-dimensional (3-D) objects onto two-dimensional (2-D) picture planes are inherently ambiguous and not invertible in the sense that the 3-D representation can be inferred from the 2-D projection. This inherent ambiguity viewers experience when viewing a picture can be clarified by making assumptions about the viewed objects. These assumptions are needed to interpret the spatial meaning of the image one may see in the picture (Gregory, 1967). One might, for example, use knowledge of the typical shape or size of an object to estimate its depicted orientation.

This kind of familiarity cue to picture interpretation does not necessarily require a complete description of the pictured object. Certain features of the object, such as parallelism or perpendicularity of some of its parts, may provide much of the information needed to calculate the depicted orientation of an object from its picture (Grunwald and Ellis, 1986). It is important, however, to realize that the ambiguity of the projection remains unresolved. It has a powerful influence on the spatial interpretation of not only pictures, but also of stereoscopically viewed, 3-D objects. Wire-frame cubes, for example, can appear to change their spatial orientation or shape and maintain nonveridical appearances even when the viewer moves. The reinterpretation due to the projective ambiguity is so strong that it overrides motion parallax cues and causes the cube to lose its perceptual rigidity when the viewer moves his or her head. Similar effects are reported to occur with more naturalistic objects, such as aircraft (McCormick, 1981).

Thus, though familiarity cues can assist interpretation of an image, the effects of the projective ambiguities persist and projected images often have multiple interpretations. The alternatives, however, are not arbitrary and can be argued to be constrained both by the viewer's experience and by such inherent principles of perceptual organization as gestalt laws. Accordingly, enumeration of the default perceptual assumptions used when viewing pictures can provide useful insights into the underlying interpretive processes.

The following description presents a new illusion of projected 3-D space. This new illusion may provide a unique exercise for models for pictorial interpretation because it involves an object made entirely of curved lines. Pirenne, who noted that correctly pictured curved shapes are associated with misjudgments of their depicted shape, has argued that curved objects such as spheres present special problems for 3-D interpretation (Pirenne, 1970; for a possible related effect also see Wallach, et al., 1956).

DESCRIPTION

We originally observed this illusion as we viewed a trace of the relative motion of one spacecraft with respect to another in nearly circular Earth orbit. When plotted in local-vertical local-horizontal coordinates, the trajectory of the craft describes, for most small perturbations of the orbit, a looped relative motion path (Thomson, 1961; Oberg, 1982). This trajectory may be described parametrically in an xy moving reference frame in which x is in the direction of the orbital velocity and y points toward the center of the Earth:

$$x = \frac{\Delta v}{V} (4 \sin \phi - 3\phi)$$

$$y = 2 \frac{\Delta v}{V} (1 - \cos \phi)$$

and where ϕ is the phase angle of the orbit. $\phi = \omega t$ when ω is the orbital rate in radians per second and t is time in seconds. $\Delta v/V$ is the relative change in orbital velocity caused by a maneuvering burn tangent to the orbit.

For the purposes of this description, this path will be approximated by the more familiar expression for a generalized cycloid: the trochoid. The trochoid describes the locus of a point on the edge of a wheel of radius b which is concentric with and attached to a smaller wheel of radius a that is rolling on a flat surface. As described below, the curve is positioned in three dimensions and has n - 1 complete loops.

$$x = a\theta - b \sin(\theta)$$

$$y = a - b \cos(\theta)$$

$$z = d$$

$$a < b ; \quad -2\pi n < \theta < 2\pi n ; \quad n = 1,2,3 \ldots$$

2

Apparent oddities in the spatial appearance of such looped paths first appeared when we viewed them obliquely and extended them outside an overlaid grid representing the orbital plane. The path, which represented a relative motion predictor for an in-plane orbital maneuver, appeared to move out of the orbital plane as if an out-of-plane maneuver was being planned. On further study we saw that the predictor path appeared to have several alternative spatial configurations. We could see it either as a looped path in the plane, or as a path weaving in and out of the orbital plane grid or, surprisingly, as a spring-like, helical shape approximately orthogonal to the grid. As may be seen from figures 1 and 2, the ambiguity of the illusion is not critically dependent on a gridded background, although the grid can provide a useful object for a reference orientation.

We have not conducted systematic experiments to determine the factors that influence the frequency of the alternative spatial interpretations of this curve, but we do have some informal observations to report. For example, when we view the configuration orthogonal to the plane, the spatial ambiguities appear to be reduced, but as the viewing angle is rotated toward parallelism with the plane, alternative interpretations become more frequent and the helical interpretation becomes more prominent (fig. 1). Also, the helical interpretation seems most obvious if the looped path is extended past the edge of the grid; indeed, that was how we first noticed the effect (fig. 2).

The perceptual instability that appears under oblique viewing of the trochoid is also noteworthy because it differs from other forms of ambiguity associated with projections of 3-D objects. For example, the various views of a wire-frame cube produced as the direction of view is varied, vary in their 3-D appearance. Most notably, a view directed along the oblique axis through the cube produces a hexagonal projection that is commonly seen as flat. This loss of 3-D interpretation of the figure may be attributed to the topology of the projection, which is quite different when the cube is viewed along the oblique axis. The perceptual ambiguity reported for the trochoid is significantly different since it does not arise from changes in the topology of the projection. A second difference between the ambiguity described in this report and previous reports is that it is not associated with a mirror reflection through a plane parallel to the viewing direction of the type associated with the two common alternative views of the Necker cube.

DISCUSSION

The appearance of the spatial ambiguities appears to require that some cue be given to the viewer that the image should be seen as a perspective projection. This assumption may be based either on apparent convergence of lines in the grid or by implied convergence lines suggested by the decreasing size of the loops. Once a perspective interpretation is assumed, the next problem is to resolve its inherent ambiguity. Clarifying this ambiguity is a major problem because, as recent results have underscored, even provision of continuous relative motion cues does not totally resolve projective ambiguities (Epstein and Park, 1986).

The spatial ambiguity of the looped path we produced would be particularly strong because the path provides few projected angles on the picture surface. As Attneave and Frost (1969) have shown, these projected angles provide critical information that can be combined with assumptions about an object's 3-D shape to recover its 3-D orientation. Nevertheless, assumptions about the curve's 3-D properties can assist in its spatial interpretation.

A key set of assumptions that must be implicitly made concerns the relative position in depth of the curve where it appears to cross itself, i.e., the front/back assignment. If both crossing lines are assumed to be at the same depth when they cross, the entire curve may seem to be confined to a plane in space. This is a common interpretation of the left part of figure 1. If one set of the crossing lines is seen consistently to be in front of the other, then the helix may be seen. Other patterns of assignment may yield other interpretations, but these do not appear to be particularly stable.

One interesting question arising about the front/back assignments concerns their independence. Proximity of one cross point, for which a front/back assumption has been made, might influence the front/back assignment of an adjacent one. To test this, one might measure the time it takes viewers to see the helix for different numbers of loops in the path. Random models of statistically independent front/back assignments like those used to model random visual search (Krendel and Wodinsky, 1960) could be used to calculate theoretical functions to compare with the data.

For example, if p_f is the probability that the left arm of a crossing is seen in front of the right arm, and on the assumption that there are n independent crossing decisions made in parallel, the probability, p_h, that all the crossings are seen as in front and are therefore consistent with a helical interpretation is

$$p_h = (p_f)^n$$

where n is the number of loops.

If the decision processes concerning the crossings occur at some average rate $1/T$, for some duration t, the number of decisions per unit time is t/T so that the probability of at least one set of decisions consistent with a helix by time t is

$$P_h(t) = 1 - (1 - p_h)^{t/T}$$

or

$$P_h(t) = 1 - e^{(t/T)\ln(1-p_h)} = 1 - e^{kt}$$

where

$$k = \frac{1}{T} \ln(1 - p_h)$$

Thus, if a perceived helix invariably follows from a consistent set of independent front/back decisions, the probability of first seeing the helical interpretation ought to be an exponential function of viewing time. Thus, the assumption of independence of front/back decisions is open to empirical test.

The proximity of adjacent crossing might, however, introduce dependencies into the front/back decisions. If proximity can be shown to be a major factor influencing front/back assignment, one could explain the more prominent helical interpretation when the plane of the path is slanted. With increasing slant, the loops will be squeezed closer in the picture plane and their increased proximity could raise the probability of all cross points having the same front/back assignment, a necessary condition for the helical interpretation.

Familiarity with coil springs may play a role too, since even a consistent front/back assignment does not totally specify the appearance of the helix. There could be infinitely many other curved paths besides the one most often seen with a decreasing radius. Perhaps this one is seen since it resembles the spring often found in a flashlight. Thus, the truth may be as Wallach and O'Connell (1953) suggested long ago, that "the three dimensional forms perceived in perspective drawing photographs, etc. are indeed a matter of previous experience." Our previous experience with everyday objects must influence the frequency with which we entertain alternative object-hypotheses about ambiguous pictured objects.

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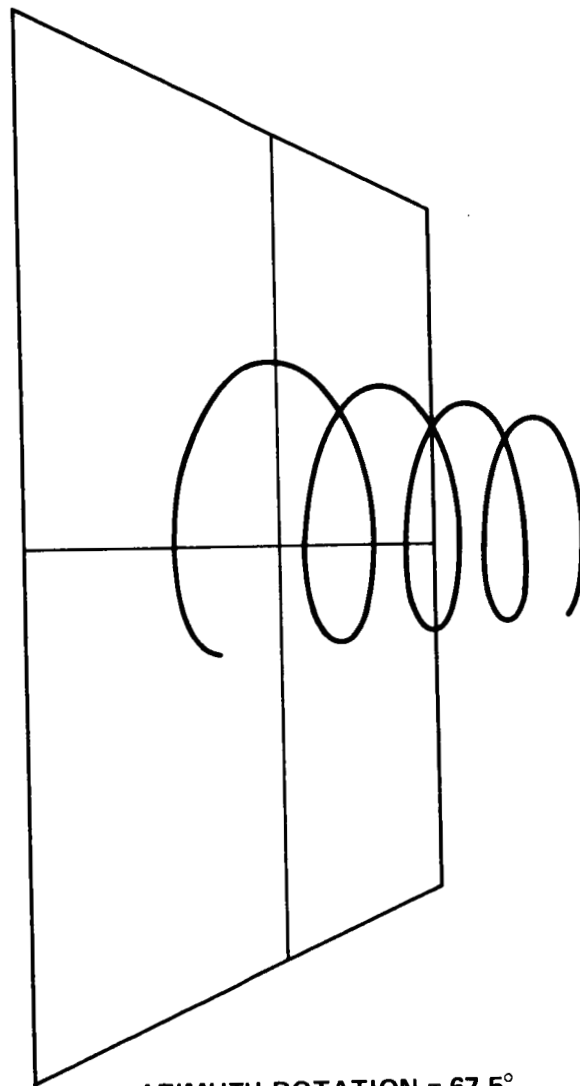
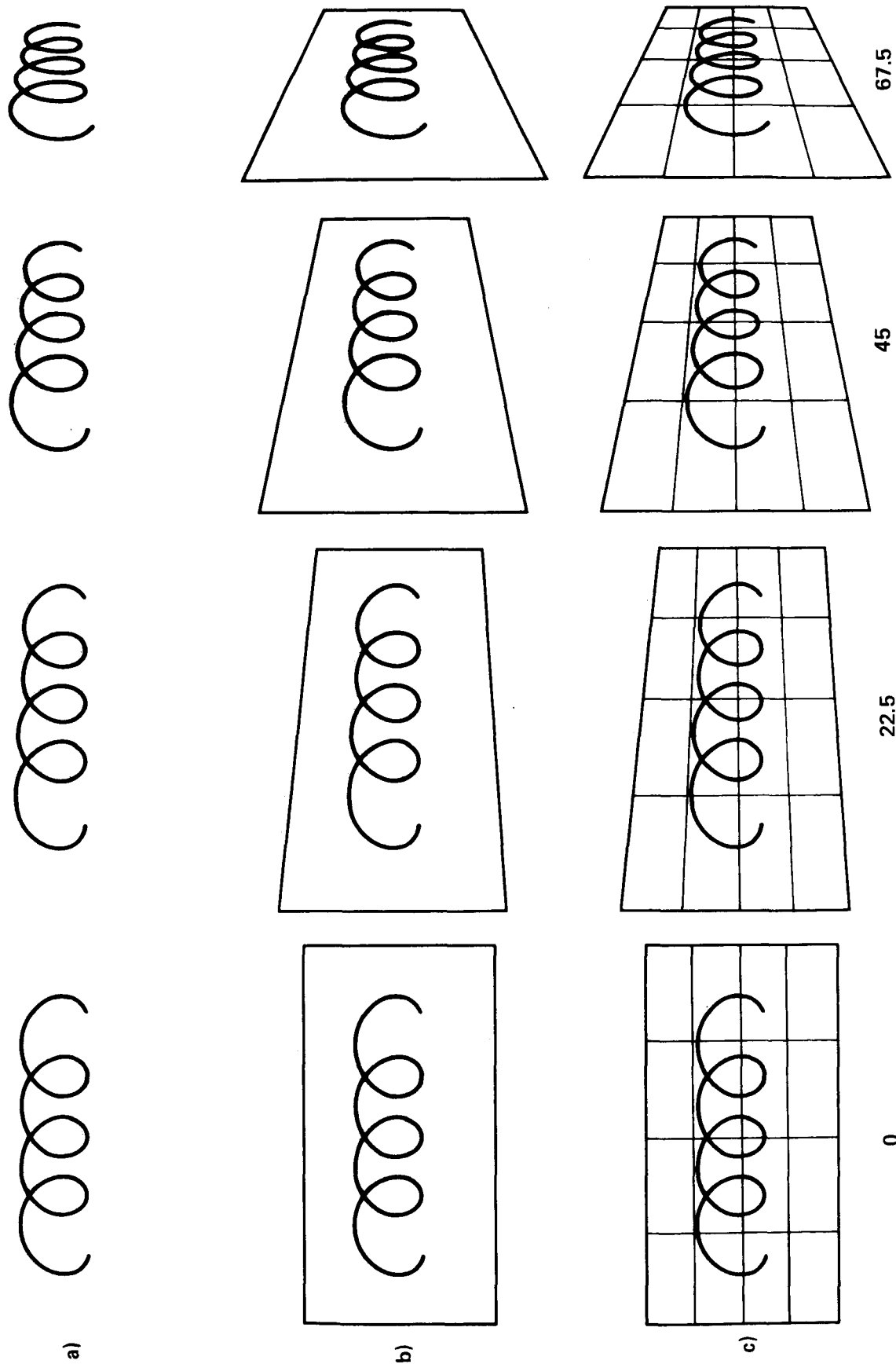AZIMUTH ROTATION = 67.5°

Figure 1.- Rotated trochoid and grid.

AZIMUTH ROTATION, deg

Figure 2.- Rotated trochoid and grids with different amounts of rotation.

8

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16. Abstract

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